Stat 156 HW #5

Due 11/12/2024 by 11:59 pm PT

Note that points will be deducted if you fail to

- Submit code for coding problems

- Assign pages to the corresponding problem in Gradescope

1. Recall from lecture that we can identify the effect of removing treatment on the whole population as

$$\mathbb{E}[Y - Y(D = 0)] = \mathbb{E}[Y] - \mathbb{E}[Y \mid Z = 0]$$

under *Relevance, Exclusion Restriction, Exchangeability* and *one-sided noncompliance* (meaning $Z = 0$ would imply $D = 0$). Also recall that under the same assumptions we can identify the effect of removing treatment on the treated as

$$\mathbb{E}[Y - Y(D = 0) \mid D = 1] = \frac{\mathbb{E}[Y] - \mathbb{E}[Y \mid Z = 0]}{\mathbb{P}(D = 1)}.$$

Prove these two results.

2. Prove that under the standard IV assumptions of relevance (Z is associated with D), exclusion restriction ($Y(Z = z, D = d) = Y(D = d)$), and exchangeable IV ($Z \perp\!\!\!\perp (D(Z = z), Y(Z = z))$), the structural equation model

$$D(Z = z) = \alpha_0 + \alpha_1 z + \nu$$
$$Y(Z = z, D = d) = \beta_0 + \beta_1 d + \epsilon$$

implies the following two-stage model of observed data

$$D = \alpha_0 + \alpha_1 Z + \nu$$
$$Y = \beta_0 + \beta_1 D + \epsilon$$

along with the three classical IV assumptions:

(a) Relevance: $cov(D, Z) \neq 0$

(b) Exclusion restriction: Z is not included in second-stage Y equation

(c) Exchangeable IV: $cov(Z, \nu) = cov(Z, \epsilon) = 0$.

3. In lecture we presented two identified forms of the coefficient B_1 in the classical IV two-stage model. Prove this equality.

$$\frac{E[Y \mid Z = 1] - E[Y \mid Z = 0]}{E[D \mid Z = 1] - E[D \mid Z = 0]} = \frac{cov(Y, Z)}{cov(D, Z)}.$$

4. Assume a classical two-stage IV model with $cov(D, Z) \neq 0$ and $cov(Z, \nu) = cov(Z, \epsilon) = 0$. Under this model, B_1 equals the coefficient in a population regression of Y on the predicted value from a population regression of D on Z. Prove this.

5. Problem 20.2 from *A First Course in Causal Inference*

6. Problem 21.7 from *A First Course in Causal Inference*

7. Problem 17.4 from *A First Course in Causal Inference*

8. Problem 17.6 from *A First Course in Causal Inference*

9. Problem 18.1 from *A First Course in Causal Inference*

10. Problem 18.2 from *A First Course in Causal Inference*